RESEARCH STATEMENT

CHEN JIANG

My main area of interest lies in algebraic geometry. I have been working on birational geometry of algebraic varieties. I am interested in topics related to the minimal model program (e.g., existence of good minimal models, non-vanishing conjecture, abundance conjecture, singularities, minimal log discrepancies) and properties of minimal varieties and Fano varieties (e.g., boundedness, birationality, geography, positivities, derived categories). I am interested in both exploring general theory in arbitrary dimensions and investigating explicit geometry and classification theory in lower dimensions.

1. Past research achievement

1.1. **Explicit birational geometry and classification theory of algebraic varieties.** One of the final goals of algebraic geometry is to classify algebraic varieties. The classification theory of algebraic curves are well-understood in the time of Riemann. The classification theory of algebraic surfaces are well-understood by the work of Enriques and Kodaira after Italian school. Now we are interested in the classification theory of algebraic varieties of dimensions 3 and higher.

1.1.1. *Noether inequality.* Noether inequality for surfaces is a milestone in the classification theory of algebraic surfaces. In [10], joint with Jungkai Chen and Meng Chen, we established the optimal Noether inequality $\mathrm{vol}(X) \geq \frac{4}{3}p_g(X) - \frac{10}{3}$ for any 3-fold X of general type with $p_g(X) \geq 11$. This means that such inequality holds for all but finitely many 3-folds of general type.

1.1.2. *Explicit birational geometry of Fano 3-folds.* I studied effective boundedness problem for Fano 3-folds which helps to understand the classificaiton of Fano 3-folds with mild singularities. In [3], working with Meng Chen, we showed that for a $\mathbb{Q}$-Fano 3-fold (resp. weak $\mathbb{Q}$-Fano 3-fold) X, $|-mK_X|$ gives a birational map for all $m \geq 39$ (resp. $m \geq 97$). Later in the joint work [13], we improved this result by showing that, for a weak $\mathbb{Q}$-Fano 3-fold X, there is a birational model Y such that $|-mK_Y|$ gives a birational map for all $m \geq 52$. In [20], we showed that for a canonical $\mathbb{Q}$-Fano 3-fold X, its anti-canonical degree $-K_X^3$ is bounded from above by 324, this is the first effective upper bound in the literature.

1.1.3. *Explicit birational geometry of Calabi–Yau 3-folds.* In [2], I showed that for a minimal 3-fold X with $K_X \equiv 0$ and a nef and big Weil divisor L, $|mL|$ and $|K_X + mL|$ give birational maps for all $m \geq 17$. In [4], I treated smooth projective irregular 3-folds (resp. 4-folds) X with $K_X \equiv 0$.

1.2. **Boundedness of Fano varieties and Calabi–Yau varieties.**

Date: August 6, 2022.

1.2.1. *BAB conjecture.* In a series of papers [1, 8, 17, 9], I studied the Borisov–Alexeev–Borisov Conjecture in dimension up to 3. In particular, I showed that the anti-canonical volumes of Fano 3-folds with ϵ-klt singularities is bounded from above, and the set of all Fano 3-folds with ϵ-klt singularities is birationally bounded. The Borisov–Alexeev–Borisov Conjecture was later proved by Birkar.

1.2.2. *LTT conjecture.* Applying boundedness results of Birkar, in [5] working with Christopher Hacon, we showed that for a uniruled variety and an ample line bundle, the sub-varieties with Fujita a-invariants bigger than that of the ambient space are contained in a proper Zariski closed subset. This confirms a conjecture by Lehmann, Tanimoto, and Tschinkel, which has applications for the geometric description of Manin's conjecture.

1.2.3. *K-semistable Fano varieties.* In [12], I used birational geometry method to show that, for any $\delta > 0$, the set of all d-dimensional K-semistable Fano varieties X with $(-K_X)^d > \delta$ is bounded. This result becomes an important step towards the K-moduli theory of Fano varieties established by Chenyang Xu and his collaborators.

It is known that any K-semistable Fano variety X has alpha invariant $\alpha(X) \geq \frac{1}{\dim X}$. In [6], I showed that $\mathbb{P}^n$ is the only K-semistable Fano manifold of dimension d with $\alpha(X) = \frac{1}{d}$.

1.2.4. *Calabi–Yau varieties.* In the joint work [14], we showed that the set of all rationally connected klt Calabi–Yau 3-folds is birationally bounded. Later in [16], I showed that the set of all non-canonical klt Calabi–Yau 3-folds is bounded modulo flops, and the global indices of klt Calabi–Yau 3-folds are bounded. In [15], we studied the boundedness of log pluricanonical representations of log Calabi–Yau pairs in dimension 2.

1.3. **Singularities in birational geometry.** I studied the minimal log discrepancies of singularities in birational geometry. In [16], I showed a gap theorem saying that there exists a constant $\delta > 0$ such that if $\mathrm{mld}(X) < 1$ for a 3-fold X, then $\mathrm{mld}(X) \leq 1 - \delta$. This confirms a special case of a conjecture of Shokurov. In [18], we confirm another conjecture of Shokurov on the singularities of the base of a conic bundle.

1.4. **Other interesting topics in biratinal geometry.** I studied Kawamata's effective nonvanishing conjecture for Calabi–Yau manifolds, namely, every nef line bundle has a global section. In [11] working with Yalong Cao, we confirmed this conjecture for all 6-dimensional hyperkähler manifolds and certain complete intersection Calabi–Yau manifolds. In [19], I proved this conjecture for all hyperkähler manifolds by showing that the coefficients of the Riemann–Roch polynomial of a hyperkähler manifold are all positive.

In [7] working with Pu Cao, we classified all torsion exceptional sheaves on certain weak del Pezzo surfaces, partially confirmed a conjecture of Okawa and Uehara.

References

[1] C. Jiang, *Bounding the volumes of singular weak log del Pezzo surfaces*, Internat. J. Math. **24** (2013), no. 13, 1350110, 27pp.

[2] C. Jiang, *On birational geometry of minimal threefolds with numerically trivial canonical divisors*, Math. Ann. **365** (2016), no. 1-2, 49–76.

[3] M. Chen, C. Jiang, *On the anti-canonical geometry of $\mathbb{Q}$-Fano threefolds*, J. Differential Geom. **104** (2016), no.1, 59–109.

[4] C. Jiang, *On irregular threefolds and fourfolds with numerically trivial canonical bundle*, Math. Z. **284** (2016), no. 1-2, 95–115.

[5] C.D. Hacon, C. Jiang, *On Fujita invariants of subvarieties of a uniruled variety*, Algebr. Geom. **4** (2017), no. 3, 304–310.

[6] C. Jiang, *K-semistable Fano manifolds with the smallest alpha invariant*, Internat. J. Math. **28** (2017), no. 6, 1750044, 9pp.

[7] P. Cao, C. Jiang, *Torsion exceptional sheaves on weak del Pezzo surfaces of Type A*, J. Algebra, **499** (2018), 583–609.

[8] C. Jiang, *On birational boundedness of Fano fibrations*, Amer. J. Math., **140** (2018), no. 5, 1253–1276.

[9] C. Jiang, *Boundedness of threefolds of Fano type with Mori fibration structures*, Math. Res. Lett. **26** (2019), no. 5, 1415–1425.

[10] J. A. Chen, M. Chen, C. Jiang, *The Noether inequality for algebraic threefolds*, Duke Math. J. **169** (2020), no. 9, 1603–1645. Addendum, Duke Math. J. **169** (2020), no. 11, 2199–2204.

[11] Y. Cao, C. Jiang, *Remarks on Kawamata's effective non-vanishing conjecture for manifolds with trivial first Chern classes*, Math. Z. 296 (2020), no. 1-2, 615–637.

[12] C. Jiang, *Boundedness of $\mathbb{Q}$-Fano varieties with degrees and alpha-invariants bounded from below*, Ann. Sci. Éc. Norm. Supér. (4) **53** (2020), no. 4, 1235–1248.

[13] M. Chen, C. Jiang, *On the anti-canonical geometry of weak $\mathbb{Q}$-fano threefolds, II*, Ann. Inst. Fourier (Grenoble) **70** (2020), no. 6, 2473–2542.

[14] W. Chen, G. Di Cerbo, J. Han, C. Jiang, R. Svaldi, *Birational boundedness of rationally connected klt Calabi–Yau 3-folds*, Adv. Math. **378** (2021), Paper No. 107541, 32 pp.

[15] C. Jiang, H. Liu, *Boundedness of log pluricanonical representations of log Calabi–Yau pairs in dimension 2*, Algebra Number Theory **15** (2021), no. 2, 545–567.

[16] C. Jiang, *A gap theorem for minimal log discrepancies of noncanonical singularities in dimension three*, J. Algebraic Geom. **30** (2021), no. 4, 759–800.

[17] C. Jiang, *Boundedness of anti-canonical volumes of singular log Fano threefolds*, Comm. Anal. Geom. **29** (2021), no. 7, 1571–1596.

[18] J. Han, C. Jiang, Y. Luo, *Shokurov's conjecture on conic bundles with canonical singularities*, Forum Math. Sigma **10** (2022), Paper No. e38, 24 pp.

[19] C. Jiang, Positivity of Riemann–Roch polynomials and Todd classes of hyperkähler manifolds", J. Algebraic Geom., https://doi.org/10.1090/jag/798.

[20] C. Jiang, Y. Zou, *An effective upper bound for anti-canonical volumes of canonical $\mathbb{Q}$-Fano threefolds*, Int. Math. Res. Not., https://doi.org/10.1093/imrn/rnac100.

Shanghai Center for Mathematical Sciences, Fudan University, Jiangwan Campus, 2005 Songhu Road, Shanghai, 200438, People's Republic of China

Email address: chenjiang@fudan.edu.cn